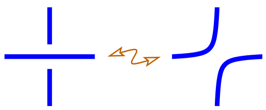
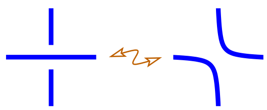
The Kauffman bracket and the Jones polynomial [Ka1]

Let L be a link diagram.

A-splitting: 

B-splitting: 

A *state* S is a choice of either A- or B-splitting at every classical crossing.

$\alpha(S) = \#(\text{of } A\text{-splittings in } S)$

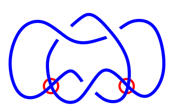
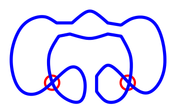
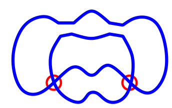
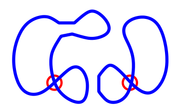
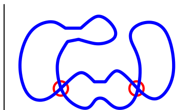
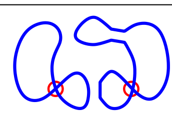
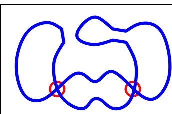
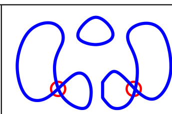
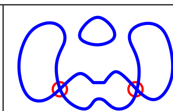
$\beta(S) = \#(\text{of } B\text{-splittings in } S)$

$\delta(S) = \#(\text{of circles in } S)$

$$[L](A,B,d) \;:=\; \sum_S A^{\alpha(S)}\, B^{\beta(S)}\, d^{\delta(S)-1}$$

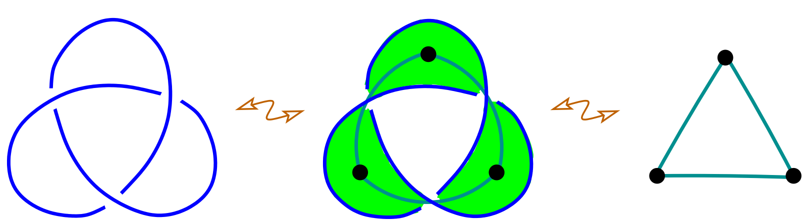
$$J_L(t) \;:= (-1)^{w(L)} t^{3w(L)/4} [L](t^{-1/4}, t^{1/4}, -t^{1/2} - t^{-1/2})$$

Example

				
(α,β,δ)	$(3,0,1)$	$(2,1,2)$	$(2,1,2)$	$(1,2,1)$
				
	$(2,1,2)$	$(1,2,1)$	$(1,2,3)$	$(0,3,2)$

$$[L] = A^3 + 3A^2Bd + 2AB^2 + AB^2d^2 + B^3d; \qquad J_L(t) = 1$$

Thistlethwaite's Theorem [Ka1] *Up to a sign and multiplication by a power of t the Jones polynomial $J_L(t)$ of an alternating link L is equal to the Tutte polynomial $T_\Gamma(-t,-t^{-1})$.*



References

[Ka1] L. H. Kauffman, *New invariants in knot theory*, Amer. Math. Monthly **95** (1988) 195–242.